"The Theory of Symmetrical Optical Objectives." By S. D. CHALMERS, B.A. (Cantab.), M.A. (Sydney), St. John's College, Cambridge. Communicated by Professor LARMOR, Sec. R.S. Received and read June 18, 1903.

This paper deals with the relations between the aberrations of a lens system, used with a front stop, and those of the compound system formed by two such systems disposed symmetrically with respect to the stop. The method of Hamilton's characteristic function is used as adapted by Maxwell, the notation employed and the expressions for aberrations of any system being those given by Thiessen.*

The optical length between the points $(x_1\ y_1\ z_1)$, $(x_2\ y_2\ z_2)$ in the medium n_{12} is $n_{12}\ \{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2\}^{\frac{1}{2}}$; which is expressible in terms of x_1, x_2, y_1, y_2 when the points lie on the surfaces $z_1=f_1\ (x, y)$ and $z_2=f_2\ (x, y)$.

If these be surfaces of revolution about the axis of z, their equations may be written $z = a_1+b_1\rho_1^2+c_1\rho_1^4+\ \ldots$, where $\rho_1^2 \equiv x_1^2+y_1^2$, and the characteristic, omitting the constant and writing χ_{12} for $-x_1x_2-y_1y_2$, is given by

$$T_{12} \equiv A_{12}\rho_1^2+B_{12}\rho_2^2+2C_{12}\chi_{12}+D_{12}\rho_1^4+E_{12}\rho_2^4+4F_{12}\chi_{12}^2+2G_{12}\rho_1^2\rho_2^2 \\ +4H_{12}\rho_1^2\chi_{12}+4J_{12}\rho_2^2\chi_{12}$$

to terms of the 4th order in x, y, where,

t_{12} representing a_2-a_1,

$$A_{12} = n_{12}\left(\frac{1}{2t_{12}}-b_1\right), \quad B_{12} = n_{12}\left(\frac{1}{2t_{12}}+b_2\right), \quad C_{12} = n_{12}\left(\frac{1}{2t_{12}}\right),$$

$$D_{12} = -n_{12}\left(\frac{1}{8t_{12}^3}-\frac{b_1}{2t_{12}^2}+c_1\right), \quad G_{12} = -n_{12}\left(\frac{1}{8t_{12}^3}+\frac{b_2-b_1}{4t_{12}^2}\right),$$

$$E_{12} = -n_{12}\left(\frac{1}{8t_{12}^3}+\frac{b_2}{2t_{12}^2}-c_2\right), \quad H_{12} = -n_{12}\left(\frac{1}{8t_{12}^3}-\frac{b_1}{4t_{12}^2}\right),$$

$$F_{12} = -n_{12}\left(\frac{1}{8t_{12}^3}\right), \quad J_{12} = -n_{12}\left(\frac{1}{8t_{12}^3}+\frac{b_2}{4t_{12}^2}\right). \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(I)}.$$

The characteristic of the media between (1) and (3) is $T_{12}+T_{23}$, where $x_2\ y_2$ are to be eliminated by the relations

$$\frac{\partial}{\partial x_2}(T_{12}+T_{23}) = 0, \qquad \frac{\partial}{\partial y_2}(T_{12}+T_{23}) = 0,$$

* 'Sitzungsberichte der K. Preuss. Akad.,' Berlin, 1890.

giving, when $B_{12}+A_{23}$ is not zero,

$$T_{13} \equiv A_{13}\rho_1^2+B_{13}\rho_3^2+2C_{13}\chi_{13}+D_{13}\rho_1^4+E_{13}\rho_3^4+4F_{13}\chi_{13}^2+2G_{13}\rho_1^2\rho_3^2 + 4H_{13}\rho_1^2\chi_{13}+4J_{13}\rho_3^2\chi_{13},$$

where

$$A_{13} = A_{12}-\frac{C_{12}^2}{A_{23}+B_{12}}, \quad B_{13} = B_{23}-\frac{C_{23}^2}{A_{23}+B_{12}}, \quad C_{13} = \frac{C_{12}\,.\,C_{23}}{A_{23}+B_{12}},$$

$$D_{13} = D_{12}-4\alpha H_{12}+2\alpha^2(G_{12}+2F_{12})-4\alpha^3 J_{12}+\alpha^4(E_{12}+D_{23}),$$

$$E_{13} = E_{23}-4\beta J_{23}+2\beta^2(G_{23}+2F_{23})-4\beta^3 H_{23}+\beta^4(E_{12}+D_{23}),$$

$$F_{13} = \alpha^2 F_{23}+\beta^2 F_{12}-2\alpha^2\beta H_{23}-2\alpha\beta^2 J_{12}+\alpha^2\beta^2(E_{12}+D_{23}),$$

$$G_{13} = \alpha^2 G_{23}+\beta^2 G_{12}-2\alpha^2\beta H_{23}-2\alpha\beta^2 J_{12}+\alpha^2\beta^2(E_{12}+D_{23}),$$

$$H_{13} = \beta H_{12}-\alpha\beta(G_{12}+2F_{12})+\alpha^3 H_{23}+3\alpha^2\beta J_{12}-\alpha^3\beta(E_{12}+D_{23}),$$

$$J_{13} = \alpha J_{23}-\alpha\beta(G_{23}+2F_{23})+\beta^3 J_{12}+3\alpha\beta^2 H_{23}-\alpha\beta^3(E_{12}+D_{23}),$$

$$\text{.................. (II),}$$

in which

$$\alpha = \frac{C_{12}}{A_{23}+B_{12}}, \qquad \beta = \frac{C_{23}}{A_{23}+B_{12}}.$$

If, however, $A_{23}+B_{12}$ is zero, (x_3y_3) is the Gaussian image of (x_1y_1) and

$$x_3 = x_1\gamma - x_3(h\rho_3^2+i\rho_2^2-2f\chi_{23})+x_2(g\rho_3^2+e\rho_2^2-2i\chi_{23}),$$

where $\gamma = -\dfrac{C_{12}}{C_{23}}$,

$$e = \frac{2}{C_{23}}(E_{12}+D_{23}), \qquad h = \frac{2}{C_{23}}\left(\frac{H_{12}}{\gamma^3}+J_{23}\right),$$

$$f = \frac{2}{C_{23}}\left(\frac{F_{12}}{\gamma^2}+F_{23}\right), \qquad i = \frac{2}{C_{23}}\left(\frac{J_{12}}{\gamma}+H_{23}\right),$$

$$g = \frac{}{C_{23}}\left(\frac{G_{12}}{\gamma^2}+G_{23}\right),$$

$$\text{.................. (III).}$$

Thus $e = 0$ is the condition for the correction of Spherical Aberration,
$i = 0$,, ,, absence of Coma,
$f = 0$,, ,, correction of Astigmatism,
$g = 0$ is the additional condition for Flatness of Field,
$h = 0$ is the condition for no Distortion.

Or if we write $2F+G \equiv 3K$ and $2f+g \equiv 3k$, then

$$k = \frac{2}{C_{23}}\left(\frac{K_{12}}{\gamma^2}+K_{23}\right),$$

and we may use the conditions $k = 0$ and $f-g = 0$ together, instead of $f = 0$ and $g = 0$.

Before applying these results to our special problem it is necessary to show that $f-g=0$ is equivalent to Petzval's condition for Flatness of Field.

From equations (II) it is evident that

$$\frac{F_{13}-G_{13}}{C_{13}^2} = \frac{F_{12}-G_{12}}{C_{12}^2} + \frac{F_{23}-G_{23}}{C_{23}^2} = \Sigma\left(\frac{F-G}{C^2}\right);$$

from (III)

$$f-g = 2\,C_{23}\left\{\frac{F_{12}-G_{12}}{C_{12}^2}+\frac{F_{23}-G_{23}}{C_{23}^2}\right\}.$$

Hence if $f-g=0$, we have

$$\Sigma\left(\frac{F-G}{C^2}\right)=0;$$

hat is, using I, we have

$$\Sigma\left\{\frac{1}{n_{12}}(b_2-b_1)\right\}=0, \quad \text{or} \quad \Sigma\left\{\left(\frac{1}{n_0}-\frac{1}{n_1}\right)\frac{1}{r}\right\}=0,$$

which is Petzval's form.

It is evident that, if this condition be satisfied for a single system, it is also satisfied for the double symmetrical system and *vice versâ.*

Application to the case of Double Symmetrical Objectives.

Consider the object plane at (0), the image due to the combined system at (3), the stop at (2), and let (1) be the plane symmetrical to (3) with regard to the stop.

Let

$$T_{12} \equiv A_{12}\rho_1^2+B_{12}\rho_2^2+2C_{12}\chi_{12}+D_{12}\rho_1^4+E_{12}\rho_2^4+4F_{12}\chi_{12}^2+2G_{12}\rho_1^2\rho_2^2 + 4H_{12}\rho_1^2\chi_{12}+4J_{12}\rho_2^2\chi_{12},$$

then from symmetry

$$T_{23} \equiv A_{12}\rho_3^2+B_{12}\rho_2^2+2C_{12}\chi_{23}+D_{12}\rho_3^4+E_{12}\rho_2^4+4F_{12}\chi_{23}^2+2G_{12}\rho_3^2\rho_2^2 + 4H_{12}\rho_3^2\chi_{23}+4J_{12}\rho_2^2\chi_{23};$$

let

$$T_{01} \equiv A_{01}\rho_0^2 + \ldots,$$

where $A_{01}=B_{01}=C_{01}=\dfrac{1}{2t_{01}}$, and

$$D_{01}=E_{01}=\ldots=-\left(\frac{1}{2t_{01}}\right)^3, \text{ assuming } n_{01}=1.$$

In the complete system

$$\gamma = -\frac{C_{02}}{C_{12}} = -\frac{C_{01}C_{12}}{(A_{12}+B_{01})\,C_{12}} = -\alpha$$

and

$$\frac{C_{01}}{A_{12}} = \frac{\alpha}{1-\alpha}.$$

$$e = \frac{2}{C_{23}}\{E_{02}+D_{23}\} = \frac{2}{C_{12}}\{E_{02}+E_{12}\}$$

$$= \frac{2}{C_{12}}\{2E_{12}-4\beta J_{12}+6\beta^2 K_{12}-4\beta^3 H_{12}+\beta^4(D_{12}+E_{01})\},$$

$$h = \frac{2}{C_{23}}\left\{\frac{H_{02}}{\gamma^3}+J_{23}\right\} = \frac{2}{C_{12}}\left\{\frac{-H_{02}}{\alpha^3}+H_{12}\right\}$$

$$= \frac{2}{C_{12}\alpha^3}\{-\beta H_{01}-3\alpha\beta K_{01}+3\alpha^2\beta J_{01}-\alpha^3\beta(D_{12}+E_{01})\}$$

$$= \frac{2\beta}{C_{12}}\{D_{12}+A_{12}{}^3\},$$

$$i = \frac{2}{C_{23}}\left\{\frac{J_{01}}{\gamma}+H_{23}\right\} = \frac{2}{C_{12}}\left\{\frac{-J_{02}}{\alpha}+J_{12}\right\}$$

$$= \frac{-2}{C_{12}\alpha}\{\alpha J_{12}-3\alpha\beta K_{12}+3\alpha^2\beta H_{12}+\beta^3 J_{01}-\alpha\beta^3(D_{12}+E_{01})-\alpha J_{12}\}$$

$$= \frac{2\beta}{C_{12}}\left\{3K_{12}-3\beta H_{12}+\beta^2 D_{12}-\beta^3\frac{1-\alpha}{\alpha}E_{01}\right\},$$

$$k = \frac{2}{C_{23}}\left\{\frac{K_{02}}{\gamma^2}+K_{23}\right\} = \frac{2}{C_{12}}\left\{\frac{K_{02}}{\alpha^2}+K_{12}\right\}$$

$$= \frac{2}{C_{12}\alpha^2}\{\beta^2 K_{01}+\alpha^2 K_{12}-2\alpha^2\beta H_{12}-2\alpha\beta^2 J_{01}+\alpha^2\beta^2(D_{12}+E_{01})+\alpha^2 K_{12}\},$$

$$= \frac{2}{C_{12}}\left\{2K_{12}-2\beta H_{12}+\beta^2 D_{12}+\left(\frac{1-\alpha}{\alpha}\right)^2 E_{01}\right\}.$$

We have $h=0$ provided $D_{12}+A_{12}{}^3=0$; but this is the condition that the single lens should be spherically corrected with regard to the centre of the stop.*

It is further evident that it is impossible to exactly satisfy $h=0$, $k=0$, and $i=0$, and it is easy to obtain the minimum residual error for a given stop and field.†

* This result is given by Professor Lummer.

† This was pointed out by H. Bruns, "Das Eikonal," 'Abh. der 2. Klasse K. Sächs. Akad.,' Leipzig, 1895. In both cases the methods employed are essentially different from the above.

When the object is at ∞, we can express the errors in terms of those of the single system with a front stop.

Let the image due to the single system be at (4), then

$$C_{34} = B_{34} = A_{34} = A_{12}, \text{ since } A_{34} + A_{12} - \frac{C_{12}^2}{B_{12}} = 0, \text{ and } A_{12} - \frac{C_{12}^2}{2B_{12}} = 0,$$

$$e_1 = \frac{2}{C_{34}} \{E_{02} + D_{24}\} = \frac{4}{C_{12}} \left\{ E_{02} + D_{23} - 4 \frac{C_{12}}{2A_{12}} H_{23} + 6 \left(\frac{C_{12}}{2A_{12}}\right)^2 K_{23} \right.$$

$$\left. - 4 \left(\frac{C_{12}}{2A_{12}}\right)^3 J_{23} + \left(\frac{C_{12}}{2A_{12}}\right)^4 (E_{23} + D_{34}) \right\}$$

$$= \frac{4}{C_{12}} \{E_{12} - 2\beta J_{12} + \tfrac{3}{2}\beta^2 K_{12} - \tfrac{1}{2}\beta^3 H_{12} + \tfrac{1}{16}\beta^4 (D_{12} - A_{12}^3)\}.$$

Again

$$\frac{K_{24}}{C_{24}^2} = \frac{K_{23}}{C_{23}^2} + \frac{K_{34}}{C_{34}^2} - \frac{2}{2A_{12}} \frac{H_{34}}{C_{34}} - \frac{2}{2A_{12}} \frac{J_{23}}{C_{23}} + \frac{1}{4A_{12}^2} (E_{23} + D_{34})$$

$$= \frac{1}{C_{12}^2} \left\{ K_{12} - \beta H_{12} + \frac{\beta^2}{4} D_{12} - \frac{\beta^2}{4} A_{12}^3 \right\}$$

therefore

$$k_1 = \frac{2}{C_{24}} \left\{ \frac{K_{02}\, C_{24}^2}{C_{02}^2} + K_{24} \right\} = \frac{4}{C_{12}} \left\{ K_{12} - \beta H_{12} + \frac{\beta^2}{4} D_{12} - \frac{\beta^2}{4} A_{12}^3 \right\}.$$

When the object is at ∞,

$$e = \frac{4}{C_{12}} \left\{ E_{12} - 2\beta J_{12} + 3\beta^2 K_{12} - 2\beta^3 H_{12} + \frac{\beta^4}{2} D_{12} \right\}$$

$$= e_1 + 6\beta^2 k_1 + \frac{4\beta^4}{C_{12}} (D_{12} + 7A_{12}^3)$$

$$h = \frac{2\beta}{C_{12}} (D_{12} + A_{12}^3),$$

$$i = \frac{6\beta}{C_{12}} \left\{ K_{12} - \beta H_{12} + \frac{\beta^2}{3} D_{12} \right\} = \frac{3\beta}{2C_{12}} k_1 + \frac{\beta^2}{2C_{12}} (D_{12} + 3A_{12}^3),$$

$$k = \frac{4}{C_{12}} \left\{ K_{12} - \beta H_{12} + \frac{\beta^2}{2} D_{12} \right\} = k_1 + \frac{\beta^2}{C_{12}} (D_{12} + A_{12}^3).$$

From these expressions it is evident that the whole system will be stigmatically corrected, if one component is so, and the additional condition $D_{12} + A_{12}^3 = 0$ is satisfied; the latter is the condition that there should be no Distortion in the whole system. It is impossible, under these conditions, to satisfy $i = 0$ exactly; but it may be shown that in symmetrical objectives, whose aperture-ratio and field correspond to those of modern Anastigmats, the combined effect of k and i will be least when $k = 0$. If also $e_1 = 0$ then $e = 0$ with sufficient approximation.

Thus with a lens 100 mm. focal length, aperture ratio F. 7, field 50°, the greatest diameter of the image of any point (defect due to i) is 0·2 mm. approx.; that of a point on the axis (defect due to e) will be approx. 0·02 mm.

For these values the effect of terms in T of higher order would be appreciable; but the results justify the practice of correcting a single component—the back one—for astigmatism and spherical aberration, *provided* due attention is paid to the securing of the condition for no distortion.

"On the Discharge of Electricity from Hot Platinum." By HAROLD A. WILSON, D.Sc., B.A., Fellow of Trinity College, Cambridge. Communicated by C. T. R. WILSON, F.R.S. Received June 3,—Read June 18, 1903.

(Abstract.)

This paper contains an account of a series of experiments on the discharge of electricity from hot platinum wires. The main object of the investigation was to determine the influence exerted by the nature of the gas in which the wire is immersed. The first part of the paper contains a short account of some of the results obtained by previous investigators. The rest of the paper is divided into the following sections:—

(1) Description of apparatus, &c.
(2) The leak in air, nitrogen and water vapour.
(3) The variation of the negative leak with the temperature.
(4) The leak in hydrogen.
(5) The leak from palladium in hydrogen.
(6) Summary of principal results.
(7) Conclusion.

The wire used was of pure platinum, and was mounted like the filament of an incandescent lamp, in a glass tube. A platinum cylinder surrounded the wire, and the current from the wire to the cylinder, with various differences of potential between them, was measured with a galvanometer. The wire was heated by passing a current through it, and its temperature was determined from its resistance.

It was found that at low pressures using a wire not specially cleaned a large negative leak could be obtained. This leak, however, was not the same on different occasions with the same wire, nor with different wires at the same temperature. The leak on first heating a wire is very large, but falls off with the time. If the wire is then left cold for some hours the leak is again large on first heating and falls off as before. If the wire is kept at a constant temperature and the leak measured for